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Event-Triggered Fault Detection of Nonlinear Networked Systems

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***Abstract*—This paper investigates the problem of fault detection for nonlinear discrete-time networked systems under an event-triggered scheme. A polynomial fuzzy fault detection filter is designed to generate a residual signal and detect faults in the system. A novel polynomial event-triggered scheme is proposed to determine the transmission of the signal. A fault detection filter is designed to guarantee that the residual system is asymptotically stable and satisfies the desired performance. Polynomial approximated membership functions obtained by Taylor series are employed for filtering analysis. Furthermore, sufficient conditions are represented in terms of sum of squares (SOS) and can be solved by SOS Tools in Matlab environment. A numerical example is provided to demonstrate the effectiveness of the proposed results.**

***Index Terms*—Nonlinear networked systems; Polynomial fuzzy model; Sum of squares; Event-triggered scheme; Fault detection.**

I. Introduction

IT is well know that the real systems are uncertain nonlinear systems [1]–[8]. Recently, some fuzzy logic control and neural control methods have been proposed to control the nonlinear systems [9]–[16]. The authors in [12] designed a novel adaptive fuzzy output feedback controller for pure-feedback interconnected nonlinear systems with unmeasured states. Recently, Takagi-Sugeno (T-S) fuzzy-model-based approach has drawn considerable attention because of its high ability on modeling nonlinear systems [17]–[20]. The model reduction problem was investigated for interval type-2 T-S fuzzy systems in [17]. The authors in [18] considered switched fuzzy output feedback control problem for nonlinear systems. Many results have been developed based on the T-S fuzzy systems for fault diagnosis [21]–[23]. To mention a few, in [23], fault estimation and fault-tolerant control for T-S fuzzy stochastic systems with sensor failures have been investigated via a novel robust observer technique. In [24], a novel fuzzy fault detection filter was proposed for T-S fuzzy systems with time-varying delays via delta operator approach. However, in the modeling process, limited system information was taken into account, which leads to more conservative results [25]. Various methods have been developed to obtain more relaxed results. A learning algorithms of cerebellar model articulation controllers to provide the robust property against outliers existing in training data was discussed in [26]. In [27], sufficient conditions are derived in terms of the matrix spectral norm of the closed-loop fuzzy system instead of the traditional fuzzy control design approaches. Moreover, the information hidden in membership functions was considered for the stability and control problems in order to get relaxed conditions in [28]–[32]. In recent years, the T-S system approach has been generalized to the polynomial T-S system approach [33], which inherits the virtues of the T-S systems approach and has two additional advantages [34]. One is that the approach can represent nonlinear systems with less number of fuzzy rules, while the other one is that the stability conditions obtained via polynomial Lyapunov functions also have less conservatism than those obtained via the well known quadratic Lyapunov functions. In order to develop these advantages, many results were investigated for polynomial T-S systems in recent years [35]–[38]. Moreover, new polynomial approximated membership functions were proposed to relax the stability conditions [39].

The communication link of the networked systems is a limited resource. Several methods have been developed to save that resource [40]–[45], in which the event generator is a critical factor [46], [47]. The role of the event generator is to determine whether or not the sampled signal should be transmitted through a predefined event-triggered scheme [48]–[51]. Because of the event generator, all the transmitted signals become more important to the control or the filtering procedures and more sensitive to faults. In addition, T-S and polynomial fuzzy approaches to networked systems bring new challenges such as dealing with the network induced delay and data packets dropout [52]–[56]. In order to increase the safety and reliability of the control signal, the fuzzy fault detection filter was designed in [57]. However, to the authors' best knowledge, there are few results on fuzzy fault detection problem under event-triggered scheme, which motivates this study.

This paper introduces a novel fault detection scheme for polynomial fuzzy discrete-time networked systems under event-triggered scheme. A polynomial fuzzy fault detection

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filter is designed under a new polynomial event-triggered scheme to guarantee the asymptotic stability of the residual system with desired performance. Polynomial approximated membership functions obtained by Taylor series are employed, and sufficient conditions are developed as SOS, which can be solved by SOSTOOLS. The main contributions of this paper can be summarized as follows: 1. We first investigate the fault detection problem for networked systems subject to event-triggered scheme. 2. A new polynomial event-triggered scheme is adopted to improve the design flexibility. 3. Polynomial T-S system approach and polynomial approximated membership functions are employed to handle the nonlinear systems and reduce the conservativeness. A numerical example is provided to demonstrate the effectiveness of the proposed methods.

The rest of this paper is organized as follows. In Section II, the residual system distributed in the network modeled as a polynomial fuzzy system is constructed, including the phenomena of event-triggered scheme. Section III proposes the approach of designing an H_∞ fault detection filter. A numerical example is exploited to show the effectiveness of the proposed approach in Section IV and we conclude this paper in Section V.

Notations: The notations used in this paper are quite standard. The symbol "$*$" represents the transposed elements of the symmetric matrix. The notation $\|H\|$ indicates the L_2 norm of matrix H defined by $\|H\| = \sqrt{\mathrm{tr}\left(H^T H\right)}$. Identity matrices with appropriate dimensions will be denoted by I. The superscripts "T" and "-1" denote the matrix transpose and inverse respectively.

II. Problem Formulation and Preliminaries

The nonlinear networked system considered in this paper is shown in Fig. 1. The output of the plant is measured

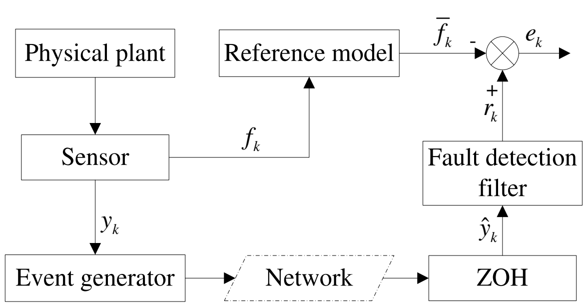

Fig. 1: The physical model of the residual system.

by the sensor and evaluated by the event generator before being communicated through the network, which saves the communication bandwidth. It is assumed that there may be faults occurred in the sensor and then the overall system is modeled as follows:

Plant Rule i : IF $\theta_1(x_k)$ is N_{i1}, and $\cdots$, and $\theta_j(x_k)$ is N_{ij}, and $\cdots$, and $\theta_p(x_k)$ is N_{ip}, THEN

$$\begin{aligned} x_{k+1} &= A_i\left(x_k\right)x_k + B_{1i}\left(x_k\right)w_k + B_{2i}\left(x_k\right)f_k, \\ y_k &= C_i\left(x_k\right)x_k + D_{1i}\left(x_k\right)w_k + D_{2i}\left(x_k\right)f_k, \end{aligned} \quad (1)$$

where $x_k \in R^n$ is the state vector, $y_k \in R^l$ is measurement output, $w_k \in R^{n_w}$ is the process disturbance belonging to $l_2\left[0, \infty\right)$, and $f_k \in R^m$ is the faults vector. Generally, it is assumed that f_k is l_2 norm bounded. $A_i\left(x_k\right)$, $B_{1i}\left(x_k\right)$, $B_{2i}\left(x_k\right)$, $C_i\left(x_k\right)$, $D_{1i}\left(x_k\right)$ and $D_{2i}\left(x_k\right)$ are polynomial system matrices. $i = 1, 2, \ldots, r$, where r is a scalar denoting the number of IF-THEN rules. $\theta_j(k)$ and N_{ij} are the premise variable and the fuzzy set, respectively. $j = 1, 2, \ldots, p$, where p is the number of the premise variables. Based on the above discussion, we obtain the global model of dynamic system as:

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^{r} h_i\left(\theta_k\right)\left[A_i\left(x_k\right)x_k\right. \\ &\qquad \left. + B_{1i}\left(x_k\right)w_k + B_{2i}\left(x_k\right)f_k\right], \\ y_k &= \sum_{i=1}^{r} h_i\left(\theta_k\right)\left[C_i\left(x_k\right)x_k\right. \\ &\qquad \left. + D_{1i}\left(x_k\right)w_k + D_{2i}\left(x_k\right)f_k\right], \end{aligned} \quad (2)$$

where $h_i(\theta_k) = \mu_i(\theta_k) \big/ \sum\limits_{i=1}^{r} \mu_i(\theta_k)$, $\mu_i(\theta_k) = \prod\limits_{j=1}^{p} N_{ij}(\theta_j(x_k))$ and $N_{ij}(\theta_j(x_k))$ is the grade of membership of $\theta_j(x_k)$ in fuzzy set N_{ij}. Usually, it is assumed that: $\mu_i(\theta_k) \geq 0$ for $i = 1, 2, \ldots, r$ and $\sum\limits_{i=1}^{r} \mu_i(\theta_k) > 0$ for all k. Therefore, $h_i\left(\theta_k\right) \geq 0$ and $\sum\limits_{i=1}^{r} h_i\left(\theta_k\right) = 1$.

One of the components of the fault detection scheme is to construct a dynamical system called the residual generator. The constructed auxiliary system takes the output of the physical plant which is assumed to be stable throughout this paper and then generates the residual signal. The residual signal is used to determine whether or not faults have occurred in the system. The following polynomial fuzzy fault detection filter is constructed to generate the residual signal.

Filter Rule i : IF $\bar{\theta}_1(x_{fk})$ is M_{i1}, and $\cdots$, and $\bar{\theta}_j(x_{fk})$ is M_{ij}, and $\cdots$, and $\bar{\theta}_q(x_{fk})$ is M_{iq}, THEN

$$\begin{aligned} x_{fk+1} &= A_{fi}\left(x_{fk}\right)x_{fk} + B_{fi}\left(x_{fk}\right)\hat{y}_k, \\ r_k &= C_{fi}\left(x_{fk}\right)x_{fk} + D_{fi}\left(x_{fk}\right)\hat{y}_k, \end{aligned} \quad (3)$$

where $x_{fk} \in R^n$ is the filter state vector, $\hat{y}_k \in R^l$ is the input vector of the filter, $r_k \in R^m$ represents residual signal, and $A_{fi}\left(x_{fk}\right)$, $B_{fi}\left(x_{fk}\right)$, $C_{fi}\left(x_{fk}\right)$ and $D_{fi}\left(x_{fk}\right)$ are the polynomial filter gain matrices to be designed. $i = 1, 2, ..., c$, the scalar c is the number of IF-THEN rules. M_{ij} and $\bar{\theta}_j(x_{fk})$ are the fuzzy set and the premise variable respectively, $j = 1, 2, \ldots, q$, where q is the number of the premise variables. Obviously, the fuzzy filter does not need to share the same premise variables and membership functions with the physical plant, which improves the design flexibility of the fuzzy filter. Similar to (2), the defuzzification of the fuzzy filter is given as:

$$\begin{aligned} x_{fk+1} &= \sum_{i=1}^{c} g_i\left(\bar{\theta}_k\right)\left[A_{fi}\left(x_{fk}\right)x_{fk} + B_{fi}\left(x_{fk}\right)\hat{y}_k\right], \\ r_k &= \sum_{i=1}^{c} g_i\left(\bar{\theta}_k\right)\left[C_{fi}\left(x_{fk}\right)x_{fk} + D_{fi}\left(x_{fk}\right)\hat{y}_k\right], \end{aligned} \quad (4)$$

where $g_i\left(\bar{\theta}_k\right)$ is the membership function, $\hat{y}_k$ is the actual input of the filter. Next, we will discuss the network introduced problems in this paper.

Remark 1: A key process of fault detection is to generate a residual signal which is sensitive to system fault. In this paper, we employ H_∞ filter to generate that residual signal. H_∞ filter can not only describe the estimated signal accurately but also suppress the disturbance effectively.

Event Detector: The purpose of introducing the event generator is to save the limited communication resource. An event-triggered scheme is adopted to determine whether or not the sampled signal should be transmitted. First, we define the difference between the current output of the plant and the last released data of the generator which is defined as $\delta_k = y_k - y_{i_k}$, where i_k, y_{i_k} denote the last released instant and the released data, respectively with $k \in [i_k, i_{k+1})$, and i_{k+1} is the next release instant of the generator. One can see that, $d = k - i_k - 1$, where k is the current time and d denotes the number of the unreleased signal between current time and last released instant.

In order to reduce the release times of the generator, the current measurement y_k satisfying $\delta_k Q\left(y_k\right)\delta_k \geq \eta y_{i_k} Q\left(y_k\right) y_{i_k}$ will be released, where $\eta > 0$ is an arbitrary scalar, and $Q\left(y_k\right)$ is an arbitrary polynomial matrix to be determined with appropriate dimensions.

In terms of the aforementioned discussions, the input of the polynomial fuzzy fault detection filter is represented as $\hat{y}_k = y_{i_k} = y_k - \delta_k$.

Remark 2: Due to the existence of the event generator, the signal released to the network channel is non-uniform signal. As shown in Fig. 1, a zero order holder (ZOH) is employed to keep the input signal of the filter as uniform discrete-time signal over all sampling instants.

Remark 3: Available results use a quadratic event detector can be found in [46]–[48]. In this paper, we use a polynomial event detector that includes the quadratic one as a special case which improves the design flexibility.

Fault Weighting System: To improve the performance, a reference residual model is usually adopted as the weighting matrix function of the fault f_k, which is represented as $\bar{f}\left(z\right) = W\left(z\right) f\left(z\right)$ [58], where $W\left(z\right)$ is given *a priori*. The choice of $W\left(z\right)$ is to impose frequency weighting on the spectrum of the fault signal for detection. A state-space realization of $W\left(z\right)$ can be

$$\begin{aligned} x_{wk+1} &= A_w x_{wk} + B_w f_k, \\ \bar{f}_k &= C_w x_{wk} + D_w f_k, \end{aligned} \tag{5}$$

where $x_{wk} \in R^k$ is the state vector, A_w, B_w, C_w, D_w are constant matrices.

Residual Evaluation: In order to facilitate the fault detection problem, the residual signal generated in the fault detection filter should be evaluated by a residual evaluation function. The prescribed evaluation function will be compared with the predefined threshold J_{th}. If the value of the evaluation function exceeds the threshold, an alarm of fault is triggered. In this paper, we adopt the following evaluation function

$$\|r\|_T \triangleq \frac{1}{T}\sqrt{\sum_{k=t_0}^{t_0+T-1} r_k^T r_k}, \;\; J_{th} \triangleq \sup_{w_k \in l_2,\; f_k = 0} \|r\|_T . \tag{6}$$

For a given threshold J_{th}, the chosen of J_{th} can refer to the discussion in [46]–[48], the generation of the alarms can be summarized as follows:

$$\begin{cases} \|r\|_T > J_{th} \Longrightarrow \text{with faults} \Longrightarrow \text{alarm} \\ \|r\|_T \leq J_{th} \Longrightarrow \text{no faults.} \end{cases}$$

Referring to (2), (4), (5) and the event-triggered scheme, we have the following residual system:

$$\begin{aligned} \varepsilon_{k+1} &= \sum_{i=1}^{r}\sum_{j=1}^{c} h_i g_j \left[\check{A}_{ij}\varepsilon_k + \check{B}_{ij}\xi_k\right], \\ e_k &= \sum_{i=1}^{r}\sum_{j=1}^{c} h_i g_j \left[\check{C}_{ij}\varepsilon_k + \check{D}_{ij}\xi_k\right], \end{aligned} \tag{7}$$

where

$$\begin{aligned} \check{A}_{ij} &= \begin{bmatrix} A_i\left(x_k\right) & 0 & 0 \\ B_{fj}\left(x_{fk}\right)E_i\left(x_k\right) & A_{fj}\left(x_{fk}\right) & 0 \\ 0 & 0 & A_w \end{bmatrix}, \\ \check{B}_i &= \begin{bmatrix} 0 & B_{1i}\left(x_k\right) & B_{2i}\left(x_k\right) \\ -B_{fj}\left(x_{fk}\right) & B_{1fj}\left(\bar{x}_k\right) & B_{2fj}\left(\bar{x}_k\right) \\ 0 & 0 & D_w \end{bmatrix}, \\ \check{C}_{ij} &= \begin{bmatrix} D_{fj}\left(x_{fk}\right)C_i\left(x_k\right) & C_{fj}\left(x_{fk}\right) & -C_w \end{bmatrix}, \\ \check{D}_{ij} &= \begin{bmatrix} -D_{fj}\left(x_{fk}\right) & D_{1fj}\left(\bar{x}_k\right) & D_{2fj}\left(\bar{x}_k\right) - D_w \end{bmatrix}, \\ B_{1fj}\left(\bar{x}_k\right) &= B_{fj}\left(x_{fk}\right)D_{1i}\left(x_k\right), \\ B_{2fj}\left(\bar{x}_k\right) &= B_{fj}\left(x_{fk}\right)D_{2i}\left(x_k\right), \\ D_{1fj}\left(\bar{x}_k\right) &= D_{fj}\left(x_{fk}\right)D_{1i}\left(x_k\right), \\ D_{2fj}\left(\bar{x}_k\right) &= D_{fj}\left(x_{fk}\right)D_{2i}\left(x_k\right), \end{aligned} \tag{8}$$

and $e_k = r_k - \bar{f}_k$, h_i and g_j denote for $h_i\left(\theta_k\right)$ and $g_j\left(\theta_k\right)$ respectively. $\varepsilon_k = \begin{bmatrix} x_k^T & x_{fk}^T & x_{wk}^T \end{bmatrix}^T$, $\xi_k = \begin{bmatrix} \delta_k^T & w_k^T & f_k^T \end{bmatrix}^T$, $k \in [i_k, i_{k+1})$.

Polynomial Approximated Membership Function: In order to reduce the conservativeness, we employ the polynomial approximated membership functions to estimate the original membership functions. Consider the system states $x\left(t\right) = \begin{bmatrix} x_1\left(t\right) & x_2\left(t\right) & \cdots & x_n\left(t\right) \end{bmatrix}^T$, $x\left(t\right) \in \Omega$, where Ω is the known bounded n dimensional state space. It is assumed that the original membership functions depend on $x_{\theta_p}\left(t\right)$, $\theta_p \in [1, n]$, $p = 1$, 2, ..., s, where s is the number of system states appeared in the original membership functions. We divide $x_{\theta_p}\left(t\right)$ into d_{θ_p} connected subregions, thus, the overall state space Ω is divided into $l = \Pi_{p=1}^{s} d_{\theta_p}$ sub-state spaces and we have $\Omega = \cup_{q=1}^{l}\Omega_q$, where Ω_q is one of the sub-state spaces. When $x_{\theta_p}\left(t\right)$ falls into one of the substate spaces, a corresponding sub-polynomial approximated membership functions will be employed.

In this paper, we will use the Taylor series approach which is represented in [59] to obtain the polynomial approximated membership functions

$$f(\mathbf{x}) = \sum_{k=0}^{\tau} \frac{1}{k!} \left(\sum_{p=1}^{s} \left(x_{\theta_p} - x_{\theta_p 0} \right) \frac{\partial}{\partial x_{\theta_p}} \right)^k f(\mathbf{x})\,|_{\mathbf{x}=\mathbf{x}_0}, \quad \tau \to \infty, \tag{9}$$

where $f(\mathbf{x})$ is an arbitrary function of $\mathbf{x}$, $\mathbf{x}_0$ is the known expansion points. $x_{\theta_i 0}$ and x_{θ_i} are one of the elements in $\mathbf{x}_0$ and $\mathbf{x}_\theta$, respectively, where $\mathbf{x}_\theta$ is an arbitrary point in the neighbourhood of $\mathbf{x}_0$. $\frac{\partial}{\partial x_{\theta_p}} f(\mathbf{x})\,|_{\mathbf{x}=\mathbf{x}_0}$ is the value of the partial derivative of $f(\mathbf{x})$ with $\mathbf{x} = \mathbf{x}_0$ which is a constant.

When $x_{\theta_i}(t)$ falls into one of the substate spaces divided by d_{θ_p}, the two endpoints of the subregion is assumed as $x_{\theta_p 1}$, $x_{\theta_p 2}$, and this assumption is valid for every $p = 1, 2, ..., s$. Then, we will employ the Taylor series expansions at every two endpoints combined by weighting functions $\upsilon_{q\theta_p p}(x_\theta)$.

Defining $m_{ij} = h_i g_j$, based on the aforementioned approach, we have the polynomial approximated membership functions $\bar{m}_{ij}$ as

$$\bar{m}_{ij} = \sum_{q=1}^{l} \sum_{\theta_1=1}^{2} \sum_{\theta_2=1}^{2} \cdots \sum_{\theta_s=1}^{2} \prod_{p=1}^{s} \upsilon_{q\theta_p p}(x_\theta)\, \chi_{ij\theta_1\theta_2\cdots\theta_s q}(\mathbf{x}), \tag{10}$$

where $\chi_{ij\theta_1\theta_2\cdots\theta_s q}(\mathbf{x})$ is the Taylor series expansion of m_{ij} with corresponding expansion points, for example, $\chi_{ij12\cdots 1q}(\mathbf{x})$ is the Taylor series expansion of m_{ij} with corresponding expansion points are $\mathbf{x} = (x_{\theta_1 1}, x_{\theta_2 2}, \cdots, x_{\theta_s 1})$ at substate space Ω_q. The weighting functions $\upsilon_{q\theta_p p}(x_\theta)$ have the following properties: $0 \le \upsilon_{q\theta_p p}(x_\theta) \le 1$ and $\upsilon_{q1p}(x_\theta) + \upsilon_{q2p}(x_\theta) = 1$, for $\theta_p = 1, 2$, $p = 1, 2, ..., s$, $\mathbf{x} \in \Omega_q$, $q = 1, 2, ..., l$. Otherwise, $\upsilon_{q\theta_p p}(x_\theta) = 0$ which lead to $\sum_{q=1}^{l} \sum_{\theta_1=1}^{2} \sum_{\theta_2=1}^{2} \cdots \sum_{\theta_s=1}^{2} \prod_{p=1}^{s} \upsilon_{q\theta_p p}(x_\theta) = 1$. Due to the effectiveness of the weighting functions $\upsilon_{q\theta_p p}(x_\theta)$, the local approximating functions $\chi_{ij\theta_1\theta_2\cdots\theta_s q}(\mathbf{x})$ are combined to approximate the original membership functions.

Problem: The problem considered in this paper is to design a polynomial fuzzy fault detection filter such that

1) The residual error system in (7) is asymptotically stable with $\bar{w}_k = 0$.

2) The residual error e_k satisfies

$$\|e\|_2 \le \gamma \|\bar{w}\|_2 \tag{11}$$

under zero-initial condition, in which γ denotes the disturbance level and $\bar{w}_k = \begin{bmatrix} w_k^T & f_k^T \end{bmatrix}^T$.

For proceeding further, the following lemma is employed.

Lemma 1: Consider a polynomial matrix $P(x_k) > 0$ and a nonsingular polynomial matrix $\Gamma(x_k) > 0$, we have

$$\lambda^2 \Gamma^T(x_k) P^{-1}(x_k) \Gamma(x_k) - \lambda\Gamma(x_k) - \lambda\Gamma^T(x_k) + P(x_k) > 0, \tag{12}$$

where λ is an arbitrary scalar to be determined.

Proof: As $P(x_k) > 0$, naturally, we have

$$(\lambda\Gamma(x_k) - P(x_k))^T P^{-1}(x_k) (\lambda\Gamma(x_k) - P(x_k)) \ge 0,$$

which implies

$$\begin{aligned} &\lambda^2 \Gamma(x_k)^T P^{-1}(x_k) \Gamma(x_k) - \lambda\Gamma(x_k)^T P^{-1}(x_k) P(x_k) \\ &- \lambda P^T(x_k) P^{-1}(x_k) \Gamma(x_k) + P^T(x_k) P^{-1}(x_k) P(x_k) \\ &\ge 0, \end{aligned}$$

then Lemma 1 holds. ■

III. Main Results

In this section, a novel idea of the PFMB approach is employed to establish the stability conditions of the residual system in (7) with the performance specified in (11).

Before proceeding further, the solution about the technologies used in this paper is presented. The SOS decomposition of multivariate polynomials is employed as the computational method. A multivariate polynomial $f(x(t))$ satisfies $f(x(t)) = \sum_{j=1}^{r} g_j(x(t))^2$, e.g., $x_1(t)^2 + 2x_1(t) + 1 = (x_1(t) + 1)^2$ is referred as sum of squares. Obviously, $f(x(t)) \ge 0$ if $f(x(t))$ is a SOS.

For a polynomial $f(x(t))$ in $x(t) \in R^n$ of degree $2d$, and $\hat{x}(x(t))$ with degree no greater than d, where $\hat{x}(x(t)) \in R^n$ is a column vector of monomials in $x(t)$. Then, a SOS with multivariate structure can be defined as $f(x(t)) = \hat{x}^T(x(t)) P \hat{x}(x(t))$, where $P \ge 0$.

In order to facilitate the analysis, the following denotations are employed. Define $\tilde{x}_k = (x_k^{s_1}, x_k^{s_2}, \ldots, x_k^{s_m})$, $S = \{s_1, s_2, \ldots, s_m\}$ represents the row indices of $B_{1i}(x_k)$ and $B_{2i}(x_k)$ whose corresponding row are both equal to zero, i.e., $B_{1i}(\tilde{x}_k) x_k = 0$ and $B_{2i}(\tilde{x}_k) x_k = 0$. In addition, $\tilde{A}_i(x_k)$ denotes a partial matrix of $A_i(x_k)$ consists of its $s_1, s_2, \ldots, s_m$ rows. Then we have $\tilde{x}_{k+1} = \tilde{A}_i(x_k) x_k$. It is assumed that $P(\tilde{x}_k)$ only depends on $\tilde{x}_k$, therefore, $P(\tilde{x}_{k+1})$ is a convex item. For brevity, in the following, P_{k+1} and P_k stand for $P(\tilde{x}_{k+1})$ and $P(\tilde{x}_k)$, respectively.

Theorem 1: The residual system in (7) with known filter gain matrices $A_{fj}(x_{fk})$, $B_{fj}(x_{fk})$, $C_{fj}(x_{fk})$, $D_{fj}(x_{fk})$ is asymptotically stable with a guaranteed H_∞ performance level γ if there exist symmetric polynomial matrices $P_k > 0$, in which $\tilde{x}_k$ is a partial vector in x_k, polynomial matrices $\Xi_{ij}(x_k)$, $\digamma_{ij}(x_k)$, and matrix G with appropriate dimensions, where $i = 1, 2, \ldots, r$, $j = 1, 2, \ldots, c$, constants $\eta > 0$, $\varepsilon > 0$, and arbitrary scalar λ, such that the following SOS optimization problem which minimizes γ subject to

$$v_1^T (P_k - \epsilon_1(\tilde{x}_k)) v_1 \text{ is SOS}, \tag{13}$$

$$v_2^T (\Xi_{ij}(x_k) - \epsilon_2(x_k)) v_2 \text{ is SOS}, \tag{14}$$

$$v_3^T (\digamma_{ij}(x_k) - \epsilon_3(x_k)) v_3 \text{ is SOS}, \tag{15}$$

$$v_4^T (\Xi_{ij}(x_k) - \Psi_{ij} - \epsilon_4(x_k)) v_4 \text{ is SOS}, \tag{16}$$

$$\begin{aligned} &-v_5^T \big[\left(\chi_{ij\theta_1\theta_2\cdots\theta_s q}(\mathbf{x}) + \underline{\alpha}_{ij} \right) \Psi_{ij} \\ &+ \left(\overline{\alpha}_{ij} - \underline{\alpha}_{ij} \right) \Xi_{ij}(x_k) \\ &+ \left(\chi_{ij\theta_1\theta_2\cdots\theta_s q}(\mathbf{x}) - \underline{\beta}_{ij} \right) \digamma_{ij}(x_k) \\ &- \epsilon_5(x_k) \big] v_5 \text{ is SOS}, \end{aligned} \tag{17}$$

has feasible solution for all i, j, θ_1, θ_2, $\cdots$, θ_s, q, where v_1, v_2, v_3, v_4, v_5 are arbitrary vectors independent of x_k, $\epsilon_1(\tilde{x}_k)$,

$\epsilon_2(x_k)$, $\epsilon_3(x_k)$, $\epsilon_4(x_k)$, $\epsilon_5(x_k)$ are nonnegative polynomial matrices with appropriate dimensions, and $\chi_{ij\theta_1\theta_2\cdots\theta_s q}(\mathbf{x})$ is defined in Section II, and $\overline{\alpha}_{ij}$, $\underline{\alpha}_{ij}$ and $\underline{\beta}_{ij}$ are presented above and below (23), respectively. In addition,

$$\begin{aligned}\Psi_{ij} &= \begin{bmatrix}\bar{\Phi}_{11} & * & * & * \\ \Phi_{21} & \Phi_{22} & * & * \\ G\check{A}_{ij} & G\check{B}_{ij} & M_{k+1} & * \\ \check{C}_{ij} & \check{D}_{ij} & 0 & -I\end{bmatrix}, \\ \Phi_{11} &= \begin{bmatrix}\eta C_i^T(x_k)Q(y_k)C_i(x_k) & * & * \\ 0 & 0 & * \\ 0 & 0 & 0\end{bmatrix},\end{aligned}$$

$$\begin{aligned}&\Phi_{22} = \begin{bmatrix}\Theta_1 & * & * \\ \Theta_2^T & \Theta_4-\gamma^2 I & * \\ \Theta_3^T & \Theta_5^T & \Theta_6-\gamma^2 I\end{bmatrix}, \\ &\Phi_{21} = \begin{bmatrix}-\eta Q(y_k)C_i(x_k) & 0 & 0 \\ \eta D_{1i}^T(x_k)Q(y_k)C_i(x_k) & 0 & 0 \\ \eta D_{2i}^T(x_k)Q(y_k)C_i(x_k) & 0 & 0\end{bmatrix}, \\ &\Theta_1 = (\eta-1)Q(y_k),\ \Theta_3 = -\eta Q(y_k)D_{2i}(x_k), \\ &\Theta_2 = -\eta Q(y_k)D_{1i}(x_k),\ \bar{\Phi}_{11} = \Phi_{11} - P_k, \\ &\Theta_4 = \eta D_{1i}^T(x_k)Q(y_k)D_{1i}(x_k), \\ &\Theta_5 = \eta D_{1i}^T(x_k)Q(y_k)D_{2i}(x_k), \\ &\Theta_6 = \eta D_{2i}^T(x_k)Q(y_k)D_{2i}(x_k), \\ &M_{k+1} = \lambda^2 P_{k+1} - \lambda G - \lambda G^T,\end{aligned}$$

in which $\check{A}_{ij}$, $\check{B}_{ij}$, $\check{C}_{ij}$ and $\check{D}_{ij}$, are represented in (8), and $Q(x_k)$ is the parameter of the event-triggered scheme to be determined, P_{k+1} is a matrix obtained by substituting the elements $\tilde{x}_k$ in matrix P_k by $\tilde{x}_{k+1} = \tilde{A}_i(x_k)x_k$, correspondingly.

Proof: Based on the Lyapunov stability theory, the polynomial Lyapunov functional is constructed as follows:

$$V_k = \varepsilon_k^T P_k \varepsilon_k. \tag{18}$$

According to the trajectories of system (7), and the problem considered in this paper, we introduce the following performance index:

$$\begin{aligned}J &= \Delta V_k + e_k^T e_k - \gamma^2 \bar{w}_k^T \bar{w}_k \\ &= \varepsilon_{k+1}^T P_{k+1}\varepsilon_{k+1} - \varepsilon_k^T P_k \varepsilon_k \\ &\quad + e_k^T e_k - \gamma^2 \bar{w}_k^T \bar{w}_k.\end{aligned} \tag{19}$$

Based on the denotations in the beginning of this section, formula (19) is converted into

$$\begin{aligned}J &= \varepsilon_{k+1}^T P_{k+1}\varepsilon_{k+1} - \varepsilon_k^T P_k\varepsilon_k + e_k^T e_k - \gamma^2 w_k^T w_k \\ &= \sum_{i=1}^{r}\sum_{j=1}^{c} m_{ij}\bar{\varepsilon}_k^T \begin{bmatrix}\check{A}_{ij} & \check{B}_{ij}\end{bmatrix}^T P_{k+1}\begin{bmatrix}\check{A}_{ij} & \check{B}_{ij}\end{bmatrix}\bar{\varepsilon}_k \\ &\quad + \sum_{i=1}^{r}\sum_{j=1}^{c} m_{ij}\bar{\varepsilon}_k \begin{bmatrix}\check{C}_{ij} & \check{D}_{ij}\end{bmatrix}^T \begin{bmatrix}\check{C}_{ij} & \check{D}_{ij}\end{bmatrix}\bar{\varepsilon}_k \\ &\quad - \varepsilon_k^T P_k \varepsilon_k - \gamma^2 \bar{w}_k^T \bar{w}_k.\end{aligned} \tag{20}$$

Considering the event-triggered communication scheme, for every $k \in [i_k, i_{k+1})$, we know $\delta_k Q(y_k)\delta_k \le \eta y_{i_k} Q(y_k) y_{i_k}$. Then the performance index represented in (20) with nonzero disturbance can be obtained:

$$\begin{aligned}J \le &\sum_{i=1}^{r}\sum_{j=1}^{c} m_{ij}\bar{\varepsilon}_k^T \begin{bmatrix}\check{A}_{ij} & \check{B}_{ij}\end{bmatrix}^T P_{k+1}\begin{bmatrix}\check{A}_{ij} & \check{B}_{ij}\end{bmatrix}\bar{\varepsilon}_k \\ &+ \sum_{i=1}^{r}\sum_{j=1}^{c} m_{ij}\bar{\varepsilon}_k^T \begin{bmatrix}\check{C}_{ij} & \check{D}_{ij}\end{bmatrix}^T \begin{bmatrix}\check{C}_{ij} & \check{D}_{ij}\end{bmatrix}\bar{\varepsilon}_k \\ &+ \sum_{i=1}^{r}\sum_{j=1}^{c} m_{ij}\bar{\varepsilon}_k^T \Phi \bar{\varepsilon}_k - \varepsilon_k^T P_k \varepsilon_k,\end{aligned} \tag{21}$$

where

$$\bar{\varepsilon}_k = \begin{bmatrix}\varepsilon_k \\ \xi_k\end{bmatrix},\ \Phi = \begin{bmatrix}\Phi_{11} & * \\ \Phi_{21} & \Phi_{22}\end{bmatrix},$$

$$\begin{aligned}\Phi_{21} &= \begin{bmatrix}-\eta Q(y_k)C_i(x_k) & 0 & 0 \\ \eta D_{1i}^T(x_k)Q(y_k)C_i(x_k) & 0 & 0 \\ \eta D_{2i}^T(x_k)Q(y_k)C_i(x_k) & 0 & 0\end{bmatrix}, \\ \Phi_{11} &= \begin{bmatrix}\eta C_i^T(x_k)Q(y_k)C_i(x_k) & * & * \\ 0 & 0 & * \\ 0 & 0 & 0\end{bmatrix}, \\ \Phi_{22} &= \begin{bmatrix}\Theta_1 & * & * \\ \Theta_2^T & \Theta_4-\gamma^2 I & * \\ \Theta_3^T & \Theta_5^T & \Theta_6-\gamma^2 I\end{bmatrix}, \\ \Theta_1 &= (\eta-1)Q(y_k),\ \Theta_2 = -\eta Q(y_k)D_{1i}(x_k), \\ \Theta_3 &= -\eta Q(y_k)D_{2i}(x_k), \\ \Theta_4 &= \eta D_{1i}^T(x_k)Q(y_k)D_{1i}(x_k), \\ \Theta_5 &= \eta D_{1i}^T(x_k)Q(y_k)D_{2i}(x_k), \\ \Theta_6 &= \eta D_{2i}^T(x_k)Q(y_k)D_{2i}(x_k).\end{aligned}$$

Referring to Lemma 1, we have $-G^T P_{k+1}^{-1} G < \lambda^2 P_{k+1} - \lambda G - \lambda G^T$, where G is an arbitrary matrix, and by Schur complement, the formula (21) can be guaranteed by

$$\Psi_{ij} < 0, \tag{22}$$

for all $i = 1, 2, \ldots, r$, $j = 1, 2, \ldots, c$, where

$$\begin{aligned}\Psi_{ij} &= \begin{bmatrix}\bar{\Phi}_{11} & \Phi_{12} & * & * \\ \Phi_{21} & \Phi_{22} & * & * \\ G\check{A}_{ij} & G\check{B}_{ij} & M_{k+1} & * \\ \check{C}_{ij} & \check{D}_{ij} & 0 & -I\end{bmatrix}, \\ \bar{\Phi}_{11} &= \Phi_{11} - P_k,\ M_{k+1} = \lambda^2 P_{k+1} - \lambda G - \lambda G^T.\end{aligned}$$

In order to obtain relaxed stability conditions, the polynomial approximated membership functions which are discussed in Section II are employed. As represented in [59], let the error between the original membership function and the polynomial approximated membership function be $\Delta m_{ij} = m_{ij} - \bar{m}_{ij}$. Denote the lower and upper bounds of Δm_{ij} as $\underline{\alpha}_{ij}$ and $\overline{\alpha}_{ij}$, respectively. Naturally, we have the following relation: $\underline{\alpha}_{ij} \le \Delta m_{ij} \le \overline{\alpha}_{ij}$, $i = 1, 2, \ldots, r$, $j = 1, 2, \ldots, c$. In addition, we introduce the slack matrices as $\Xi_{ij}(x_k)$ with appropriate dimensions which satisfy $0 < \Xi_{ij}(x_k) = \Xi_{ij}^T(x_k)$ and $\Xi_{ij}(x_k) \ge \Psi_{ij}$ for all i, j. Thus, the inequality in (22) can be rewritten as

$$\begin{aligned}
&\sum_{i=1}^{r}\sum_{j=1}^{c} m_{ij}\Psi_{ij} = \sum_{i=1}^{r}\sum_{j=1}^{c}\left(\bar{m}_{ij}+\Delta m_{ij}\right)\Psi_{ij} \\
= &\sum_{i=1}^{r}\sum_{j=1}^{c}\left[\left(\bar{m}_{ij}+\underline{\alpha}_{ij}\right)\Psi_{ij}+\left(\Delta m_{ij}-\underline{\alpha}_{ij}\right)\Psi_{ij}\right] \\
\leq &\sum_{i=1}^{r}\sum_{j=1}^{c}\left[\left(\bar{m}_{ij}+\underline{\alpha}_{ij}\right)\Psi_{ij}+\left(\overline{\alpha}_{ij}-\underline{\alpha}_{ij}\right)\Xi_{ij}\left(x_k\right)\right] \\
< & \; 0. \qquad (23)
\end{aligned}$$

Furthermore, we also introduce the lower bound of $\bar{m}_{ij}$ which is denoted as $\underline{\beta}_{ij}$ to relax the stability conditions. Meanwhile, slack matrices $\digamma_{ij}\left(x_k\right)$ satisfying $0<\digamma_{ij}\left(x_k\right)=\digamma_{ij}^T\left(x_k\right)$ are employed which implies the following inequality

$$\begin{aligned}
&\sum_{i=1}^{r}\sum_{j=1}^{c}\left[\left(\bar{m}_{ij}+\underline{\alpha}_{ij}\right)\Psi_{ij}+\left(\overline{\alpha}_{ij}-\underline{\alpha}_{ij}\right)\Xi_{ij}\left(x_k\right)\right. \\
&\left.+\left(\bar{m}_{ij}-\underline{\beta}_{ij}\right)\digamma_{ij}\left(x_k\right)\right]<0. \qquad (24)
\end{aligned}$$

Recalling $\bar{m}_{ij} = \sum_{q=1}^{l}\sum_{\theta_1=1}^{2}\sum_{\theta_2=1}^{2}\cdots\sum_{\theta_s=1}^{2}\prod_{p=1}^{s}\upsilon_{q\theta_p p}\left(x_\theta\right)\times \chi_{ij\theta_1\theta_2\cdots\theta_s q}\left(\mathbf{x}\right)$ and the properties of weighting function $\upsilon_{q\theta_p p}\left(x_\theta\right)$, we know that $J<0$ can be guaranteed by

$$\begin{aligned}
&\sum_{i=1}^{r}\sum_{j=1}^{c}\left(\chi_{ij\theta_1\theta_2\cdots\theta_s q}\left(\mathbf{x}\right)+\underline{\alpha}_{ij}\right)\Psi_{ij} \\
&+\left(\overline{\alpha}_{ij}-\underline{\alpha}_{ij}\right)\Xi_{ij}\left(x_k\right) \\
&+\left(\chi_{ij\theta_1\theta_2\cdots\theta_s q}\left(\mathbf{x}\right)-\underline{\beta}_{ij}\right)\digamma_{ij}\left(x_k\right)<0, \qquad (25)
\end{aligned}$$

for all θ_1, θ_2, $\cdots$, θ_s, q.

It can be seen from (25) that $J<0$ which implies

$$\Delta V_k + e_k^T e_k - \gamma^2 \bar{w}_k^T \bar{w}_k < 0.$$

In addition, because of $e_k^T e_k \geq 0$, under $w_k \equiv 0$, then $\Delta V_k < 0$. Besides, summing up on both sides for all k, where $k=0$, 1, 2, $\cdots$, ∞, then we obtain:

$$\varepsilon_\infty^T P \varepsilon_\infty - \varepsilon_0^T P \varepsilon_0 + \|e\|_2^2 - \gamma^2\|\bar{w}\|_2^2 \leq 0.$$

Considering zero initial condition and $\varepsilon_\infty^T P\varepsilon_\infty > 0$, we have

$$\|e\|_2 - \gamma\|\bar{w}\|_2 < 0, \qquad (26)$$

that is, the asymptotic stability of filtering error system (7) with an H_∞ performance being guaranteed. ■

It is known that, based on Theorem 1, if the filter gain matrices $(A_{fj}(x_{fk})$, $B_{fj}(x_{fk})$, $C_{fj}(x_{fk})$, $D_{fj}(x_{fk}))$ are given, the conditions (13)–(17) can be solved via SOSTOOLS. However, since the main purpose of this paper is to design the fault detection filter which concerned with the determination of the filter gain matrices, so that the above conditions are nonconvex. In order to deal with the nonconvex parts, we develop the following theorem.

Theorem 2: The residual system in (7) is said to be asymptotically stable with a guaranteed H_∞ performance level γ, if there exist symmetric polynomial matrices $P_k>0$, in which $\tilde{x}_k$ is a partial vector in x_k, polynomial matrices $\Xi_{ij}\left(x_k\right)$, $\digamma_{ij}\left(x_k\right)$, and matrices G_1, G_3, G_5 with appropriate dimensions, where $i=1$, 2, $\ldots$, r, $j=1$, 2, $\ldots$, c, constants $\eta>0$, $\varepsilon>0$, and arbitrary scalars a, b and λ, such that the following SOS optimization problem which minimizes γ subject to

$$v_1^T\left(P_k-\epsilon_1\left(\tilde{x}_k\right)\right)v_1 \text{ is SOS}, \qquad (27)$$

$$v_2^T\left(\Xi_{ij}\left(x_k\right)-\epsilon_2\left(x_k\right)\right)v_2 \text{ is SOS}, \qquad (28)$$

$$v_3^T\left(\digamma_{ij}\left(x_k\right)-\epsilon_3\left(x_k\right)\right)v_3 \text{ is SOS}, \qquad (29)$$

$$v_4^T\left(\Xi_{ij}\left(x_k\right)-\bar{\Psi}_{ij}-\epsilon_4\left(x_k\right)\right)v_4 \text{ is SOS}, \qquad (30)$$

$$\begin{aligned}
&-v_5^T\left[\left(\chi_{ij\theta_1\theta_2\cdots\theta_s q}\left(\mathbf{x}\right)+\underline{\alpha}_{ij}\right)\bar{\Psi}_{ij}\right. \\
&+\left(\overline{\alpha}_{ij}-\underline{\alpha}_{ij}\right)\Xi_{ij}\left(x_k\right) \\
&+\left(\chi_{ij\theta_1\theta_2\cdots\theta_s q}\left(\mathbf{x}\right)-\underline{\beta}_{ij}\right)\digamma_{ij}\left(x_k\right) \\
&\left.-\epsilon_5\left(x_k\right)\right]v_5 \text{ is SOS}, \qquad (31)
\end{aligned}$$

has feasible solution for all i, j, θ_1, θ_2, $\cdots$, θ_s, q, where v_1, v_2, v_3, v_4, v_5 are arbitrary vectors independent of x_k, and $\epsilon_1\left(\tilde{x}_k\right)$, $\epsilon_2\left(x_k\right)$, $\epsilon_3\left(x_k\right)$, $\epsilon_4\left(x_k\right)$, $\epsilon_5\left(x_k\right)$ are nonnegative polynomial matrices with appropriate dimensions, $\chi_{ij\theta_1\theta_2\cdots\theta_s q}\left(\mathbf{x}\right)$ is defined in Section II, and $\overline{\alpha}_{ij}$, $\underline{\alpha}_{ij}$ and $\underline{\beta}_{ij}$ are presented above and below (23), respectively. In addition,

$$\begin{aligned}
\bar{\Psi}_{ij} &= \begin{bmatrix} \bar{\Phi}_{11} & * & * & * \\ \Phi_{21} & \Phi_{22} & * & * \\ \bar{A}_{ij} & \bar{B}_{ij} & M_{k+1} & * \\ \check{C}_{ij} & \check{D}_{ij} & 0 & -I \end{bmatrix}, \\
\Phi_{21} &= \begin{bmatrix} -\eta Q\left(y_k\right)C_i\left(x_k\right) & 0 & 0 \\ \eta D_{1i}^T\left(x_k\right)Q\left(y_k\right)C_i\left(x_k\right) & 0 & 0 \\ \eta D_{2i}^T\left(x_k\right)Q\left(y_k\right)C_i\left(x_k\right) & 0 & 0 \end{bmatrix}, \\
\bar{\Phi}_{11} &= \Phi_{11}-P\left(\tilde{x}_k\right), \; M_{k+1}=\lambda^2 P_{k+1}-\lambda G-\lambda G^T, \\
\Phi_{11} &= \begin{bmatrix} \eta C_i^T\left(x_k\right)Q\left(y_k\right)C_i\left(x_k\right) & * & * \\ 0 & 0 & * \\ 0 & 0 & 0 \end{bmatrix}, \\
\bar{A}_{ij} &= \begin{bmatrix} \bar{\Upsilon}_1 & a\bar{A}_{fj}\left(x_{fk}\right) & 0 \\ \bar{\Upsilon}_2 & b\bar{A}_{fj}\left(x_{fk}\right) & 0 \\ 0 & 0 & G_5 A_w \end{bmatrix}, \\
\Phi_{22} &= \begin{bmatrix} \Theta_1 & * & * \\ \Theta_2^T & \Theta_4-\gamma^2 I & * \\ \Theta_3^T & \Theta_5^T & \Theta_6-\gamma^2 I \end{bmatrix}, \\
\bar{B}_{ij} &= \begin{bmatrix} -a\bar{B}_{fj}\left(x_{fk}\right) & \bar{\Upsilon}_3 & \bar{\Upsilon}_5 \\ -b\bar{B}_{fj}\left(x_{fk}\right) & \bar{\Upsilon}_4 & \bar{\Upsilon}_6 \\ 0 & 0 & G_5 D_w \end{bmatrix},
\end{aligned}$$

in which

$$\begin{aligned}
\Theta_1 &= (\eta-1)\,Q(y_k)\,,\ \Theta_2=-\eta Q(y_k)\,D_{1i}(x_k)\,,\\
\Theta_3 &= -\eta Q(y_k)\,D_{2i}(x_k)\,,\\
\Theta_4 &= \eta D_{1i}^T(x_k)\,Q(y_k)\,D_{1i}(x_k)\,,\\
\Theta_5 &= \eta D_{1i}^T(x_k)\,Q(y_k)\,D_{2i}(x_k)\,,\\
\Theta_6 &= \eta D_{2i}^T(x_k)\,Q(y_k)\,D_{2i}(x_k)\,,\\
\bar{\Upsilon}_1 &= G_1A_i(x_k)+a\bar{B}_{fj}(x_{fk})\,C_i(x_k)\,,\\
\bar{\Upsilon}_2 &= G_3A_i(x_k)+b\bar{B}_{fj}(x_{fk})\,C_i(x_k)\,,\\
\bar{\Upsilon}_3 &= G_1B_{1i}(x_k)+a\bar{B}_{fj}(x_{fk})\,D_{1i}(x_k)\,,\\
\bar{\Upsilon}_4 &= G_3B_{1i}(x_k)+b\bar{B}_{fj}(x_{fk})\,D_{1i}(x_k)\,,\\
\bar{\Upsilon}_5 &= G_1B_{2i}(x_k)+a\bar{B}_{fj}(x_{fk})\,D_{2i}(x_k)\,,\\
\bar{\Upsilon}_6 &= G_3B_{2i}(x_k)+b\bar{B}_{fj}(x_{fk})\,D_{2i}(x_k)\,.
\end{aligned}$$

and $\check{C}_{ij}$, $\check{D}_{ij}$, are represented in (8), and $Q(y_k)$ is the parameter of the event-triggered scheme to be determined, P_{k+1} is a matrix obtained by substituting the elements $\tilde{x}_k$ in matrix P_k by $\tilde{x}_{k+1}=\tilde{A}_i(x_k)\,x_k$, correspondingly. And the filter gain matrices can be calculated as $A_{fj}(x_{fk}) = G_3^{-1}\bar{A}_{fj}(x_{fk})$, $B_{fj}(x_{fk}) = G_3^{-1}\bar{B}_{fj}(x_{fk})$, $C_{fj}(x_{fk}) = C_{fj}(x_{fk})$, $D_{fj}(x_{fk})=D_{fj}(x_{fk})$.

Proof: In order to deal with the nonconvex parts discussed in Theorem 1, we denote the arbitrary matrix G is in the form of

$$G=\begin{bmatrix} G_1 & G_2 & 0\\ G_3 & G_4 & 0\\ 0 & 0 & G_5\end{bmatrix},$$

then we have

$$G\check{A}_{ij}=\begin{bmatrix} \Upsilon_1 & G_2A_{fj}(x_{fk}) & 0\\ \Upsilon_2 & G_4A_{fj}(x_{fk}) & 0\\ 0 & 0 & G_5A_w\end{bmatrix},$$

$$G\check{B}_{ij}=\begin{bmatrix} -G_2B_{fj}(x_{fk}) & \Upsilon_3 & \Upsilon_5\\ -G_4B_{fj}(x_{fk}) & \Upsilon_4 & \Upsilon_6\\ 0 & 0 & G_5D_w\end{bmatrix},$$

in which

$$\begin{aligned}
\Upsilon_1 &= G_1A_i(x_k)+G_2B_{fj}(x_{fk})\,C_i(x_k)\,,\\
\Upsilon_2 &= G_3A_i(x_k)+G_4B_{fj}(x_{fk})\,C_i(x_k)\,,\\
\Upsilon_3 &= G_1B_{1i}(x_k)+G_2B_{fj}(x_{fk})\,D_{1i}(x_k)\,,\\
\Upsilon_4 &= G_3B_{1i}(x_k)+G_4B_{fj}(x_{fk})\,D_{1i}(x_k)\,,\\
\Upsilon_5 &= G_1B_{2i}(x_k)+G_2B_{fj}(x_{fk})\,D_{2i}(x_k)\,,\\
\Upsilon_6 &= G_3B_{2i}(x_k)+G_4B_{fj}(x_{fk})\,D_{2i}(x_k)\,.
\end{aligned}$$

Denote $G_2=aG_3$, $G_4=bG_3$, where a, b are arbitrary scalars, and let $\bar{A}_{fj}(x_{fk}) = G_3A_{fj}(x_{fk})$, $\bar{B}_{fj}(x_{fk}) = G_3B_{fj}(x_{fk})$, one can see that $J<0$ can be guaranteed by

$$\bar{\Psi}_{ij}=\begin{bmatrix} \bar{\Phi}_{11} & * & * & *\\ \Phi_{21} & \Phi_{22} & * & *\\ \bar{A}_{ij} & \bar{B}_{ij} & M_{k+1} & *\\ \check{C}_{ij} & \check{D}_{ij} & 0 & -I\end{bmatrix}<0,$$

where

$$\begin{aligned}
\bar{A}_{ij} &= \begin{bmatrix} \bar{\Upsilon}_1 & a\bar{A}_{fj}(x_{fk}) & 0\\ \bar{\Upsilon}_2 & b\bar{A}_{fj}(x_{fk}) & 0\\ 0 & 0 & G_5A_w\end{bmatrix},\\
\bar{B}_{ij} &= \begin{bmatrix} -a\bar{B}_{fj}(x_{fk}) & \bar{\Upsilon}_3 & \bar{\Upsilon}_5\\ -b\bar{B}_{fj}(x_{fk}) & \bar{\Upsilon}_4 & \bar{\Upsilon}_6\\ 0 & 0 & G_5D_w\end{bmatrix},\\
\bar{\Upsilon}_1 &= G_1A_i(x_k)+a\bar{B}_{fj}(x_{fk})\,C_i(x_k)\,,\\
\bar{\Upsilon}_2 &= G_3A_i(x_k)+b\bar{B}_{fj}(x_{fk})\,C_i(x_k)\,,\\
\bar{\Upsilon}_3 &= G_1B_{1i}(x_k)+a\bar{B}_{fj}(x_{fk})\,D_{1i}(x_k)\,,\\
\bar{\Upsilon}_4 &= G_3B_{1i}(x_k)+b\bar{B}_{fj}(x_{fk})\,D_{1i}(x_k)\,,\\
\bar{\Upsilon}_5 &= G_1B_{2i}(x_k)+a\bar{B}_{fj}(x_{fk})\,D_{2i}(x_k)\,,\\
\bar{\Upsilon}_6 &= G_3B_{2i}(x_k)+b\bar{B}_{fj}(x_{fk})\,D_{2i}(x_k)\,.
\end{aligned}$$

Similar to the processes from (23) to (26) in Theorem 1, Theorem 2 can be guaranteed, and the proof is completed. ■

To have a feasible solution, we now propose the following algorithm to obtain the appropriate parameters:

Step 1: Give the nonlinear networked system and obtain the polynomial T-S fuzzy model.

Step 2: Obtain the polynomial T-S system of the system in Step 1 based on the approach in [33].

Step 3: Design the polynomial fuzzy fault detection filter for the system in Step 2.

Step 4: Obtain the polynomial approximated membership functions according to the process in Section III.

Step 5: Give the predefined parameter η, and solve the solution which minimize γ subject to (27)-(31) to obtain the filter gains $A_{fj}(x_{fk})$, $B_{fj}(x_{fk})$, $C_{fj}(x_{fk})$ and $D_{fj}(x_{fk})$.

Remark 4: The above algorithm provides the implementation process of our method in practical systems. It can address fault detection problems for a class of nonlinear networked systems with less waste of network resources. Although the method is based on the assumption of specific network environment, it can be easily generalized into other situations.

IV. Illustrative Example

In this section, to illustrate the proposed method, a simulation example is given. A two-rule polynomial fuzzy system is used to represent the nonlinear system. The relevant system parameters are given as follows:

$$\begin{aligned}
A_1 &= \begin{bmatrix} 1.5-0.2\,(x_1-2)^2 & 0.5\\ -0.56 & 0.8\end{bmatrix},\\
A_2 &= \begin{bmatrix} -1.5+0.5\,(x_2+1)^2 & 1.5\\ 0.42 & -2\end{bmatrix},\\
B_{11} &= \begin{bmatrix} 1\\ 0\end{bmatrix},\ B_{12}=\begin{bmatrix} 1\\ 0\end{bmatrix},\\
B_{21} &= \begin{bmatrix} 0.5\\ 0\end{bmatrix},\ B_{22}=\begin{bmatrix} 0.5\\ 0\end{bmatrix},\\
C_1 &= \begin{bmatrix} 1 & 0.6\end{bmatrix},\ C_2=\begin{bmatrix} 1 & 0.6\end{bmatrix},\\
D_{11} &= 0.5,\ D_{12}=0.5,\ D_{21}=1,\ D_{22}=1,
\end{aligned}$$

where the membership functions of the polynomial fuzzy plant are given as $h_1(\theta_k)=1-\frac{1}{1+e^{-x_1+1}}\in[0,1]$, and $h_2(\theta_k) = 1-h_1(\theta_k)$.

The parameters of the fault weighting system are supposed to be $A_w = 0.1$, $B_w = 0.25$, $C_w = 0.2$, $D_w = 0.5$, respectively. In addition, we also choose the membership functions of the fault detection filter as $g_1\left(\bar{\theta}_k\right) = e^{-\frac{x_{f1}^2}{2}} \in [0,1]$, and $g_2\left(\bar{\theta}_k\right) = 1 - g_1\left(\bar{\theta}_k\right)$.

Referring to the polynomial approximated membership functions in (9), the order τ is chosen as 1, and considering $x_1 \in [-2,2]$, we choose the expansion points as $\{(x_1, x_{f1})\,|x_1 \in \{-2,\ 0,\ 2\},\ x_{f1} \in \{-2,\ 0,\ 2\}\}$. Then the local approximating functions $\chi_{ij\theta_1\theta_2\cdots\theta_s q}(\mathbf{x})$ can be worked out. In addition, the weighting functions $\upsilon_{q11}(x_1)$, $\upsilon_{q21}(x_1)$, $\upsilon_{q12}(x_1)$ and $\upsilon_{q22}(x_1)$ are assumed to be $\upsilon_{q11}(x_1) = \frac{x_{12}-x_1}{x_{12}-x_{11}}$, $\upsilon_{q21}(x_1) = 1 - \upsilon_{q11}(x_1)$, $\upsilon_{q12}(x_{f1}) = \frac{x_{f12}-x_{f1}}{x_{f12}-x_{f11}}$, $\upsilon_{q22}(x_1) = 1 - \upsilon_{q12}(x_{f1})$, respectively. Thus, the polynomial approximated membership functions $\bar{m}_{ij}$ can be calculated based on (10). Setting x_1 and x_{f1} as a series of compact points, and computing the difference between the original membership functions and the polynomial approximated membership functions, we obtain the lower and upper bounds of the approximation error: $\overline{\alpha}_{11} = 0.1182$, $\overline{\alpha}_{12} = 0.0545$, $\overline{\alpha}_{21} = 0.0725$, $\overline{\alpha}_{22} = 0.0249$, $\underline{\alpha}_{11} = -0.0289$, $\underline{\alpha}_{12} = -0.0940$, $\underline{\alpha}_{21} = -0.0392$, $\underline{\alpha}_{22} = -0.0708$. Similarly, the lower bounds of $\bar{m}_{ij}$ is obtained as: $\underline{\beta}_{11} = 0.0380$, $\underline{\beta}_{12} = 0$, $\underline{\beta}_{21} = 0.0064$, $\underline{\beta}_{22} = 0$. The degree of the slack matrices $\Xi_{ij}(x_k)$ and $F_{ij}(x_k)$ are assumed as 0.

In addition, the arbitrary scalars λ, a, b are chosen as 1, 2, 0.8, respectively, and the external disturbance is assumed to be:

$$w_k = \begin{cases} \frac{1}{1+0.25dk}, & 10 \le k \le 70, \\ 0, & \text{otherwise}, \end{cases} \tag{32}$$

where the sampling period $d = 0.01$ s. Meanwhile, the faults are supposed to

$$f_k = \begin{cases} 1, & 30 \le k \le 60, \\ 0, & \text{otherwise}. \end{cases}$$

Using the SOS Tools in Matlab, according to Theorem 2, with the assumption of the polynomial filter gain matrices of degree 0 in x_{fk} (constant matrices), in the context of $\eta = 0.2$, the filter gain matrices can be calculated as follows:

$$\begin{aligned}
A_{f1}(x_{fk}) &= \begin{bmatrix} -0.2204 & 0.1507 \\ -0.1292 & -0.2286 \end{bmatrix}, \\
A_{f2}(x_{fk}) &= \begin{bmatrix} -0.2203 & 0.1522 \\ -0.1294 & -0.2293 \end{bmatrix}, \\
B_{f1}(x_{fk}) &= \begin{bmatrix} -0.2620 \\ 0.0061 \end{bmatrix},\ B_{f2}(x_{fk}) = \begin{bmatrix} -0.2621 \\ 0.0055 \end{bmatrix}, \\
C_{f1}(x_{fk}) &= \begin{bmatrix} 0.1166 & 0.3511 \end{bmatrix} \times 10^{-3}, \\
C_{f2}(x_{fk}) &= \begin{bmatrix} 0.1166 & 0.3512 \end{bmatrix} \times 10^{-3}, \\
D_{f1}(x_{fk}) &= 0.1753 \times 10^{-3},\ D_{f2}(x_{fk}) = 0.1753 \times 10^{-3},
\end{aligned}$$

meanwhile, the parameter of the polynomial event detector is obtained as

$$\begin{aligned}
Q(y_k) &= 0.8047 \times 10^{-5} x_1^2 - 0.8194 \times 10^{-6} x_1 x_2 \\
&\quad + 0.4238 \times 10^{-6} x_1 + 0.1807 \times 10^{-6} x_2^2 \\
&\quad + 0.3837 \times 10^{-6} x_2 + 0.7846 \times 10^{-4}.
\end{aligned}$$

TABLE I: Minimum γ of polynomial and quadratic event-triggered scheme for different η

η	0.1	0.2	0.3	0.4	0.5
γ_1	0.6255	0.6289	0.6660	0.6532	0.6399
γ_2	0.6208	0.6284	0.6555	0.6447	0.6354

Besides, the H_∞ disturbance attenuation level can be minimized as $\gamma = 0.6284$. Fig. 2 demonstrates the release instants of the event detector. In the simulation time, only 31 times are triggered which is much less than the time-triggered scheme (100 times). Fig. 3 shows the effectiveness of event-triggered scheme to the measurement output.

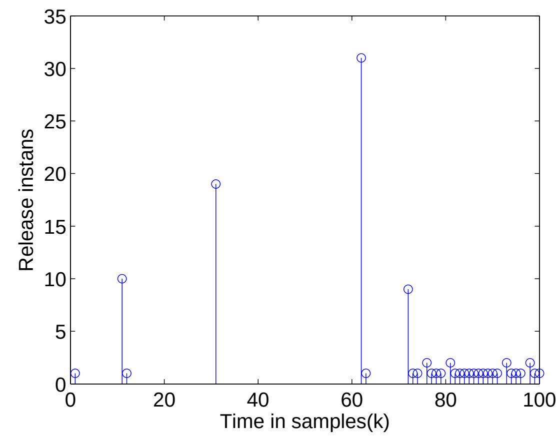

Fig. 2: The release instants.

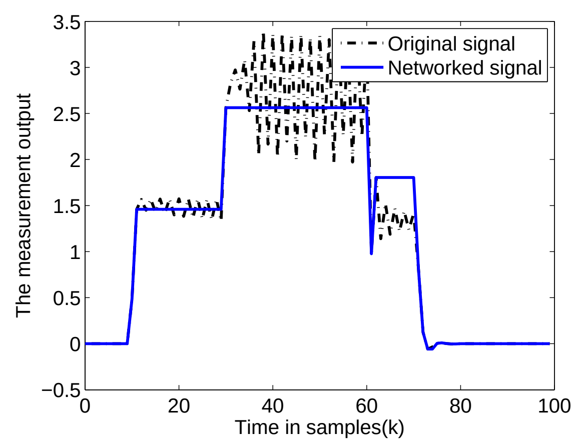

Fig. 3: The measured control output.

For different values of η, the variations of the H_∞ disturbance attenuation level γ for both quadratic and polynomial event-triggered scheme are shown in Table I, where γ_1 represents the disturbance attenuation level in the context of quadratic event-triggered scheme and γ_2 represents the polynomial one. With the variation of value η, the disturbance attenuation levels obtained by polynomial event-triggered scheme are always smaller than that obtained by quadratic event-triggered scheme which imply a higher performance.

Under zero initial condition, Figs. 4 and 5 show the residual response and the residual evaluation function response with the disturbance $w_k = 0$, respectively, and Figs. 6 and 7 show the same responses with the above mentioned disturbance inputs. One can see that, the residual can not only detect the fault in time, but also identifies the fault from the influence of disturbance w_k.

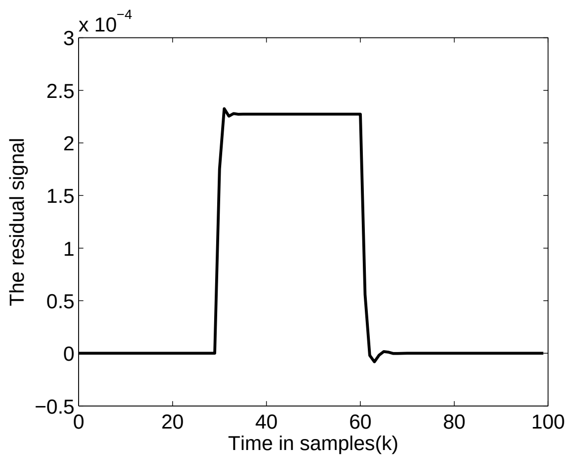

Fig. 4: Residual response of the nominal system with zero w_k.

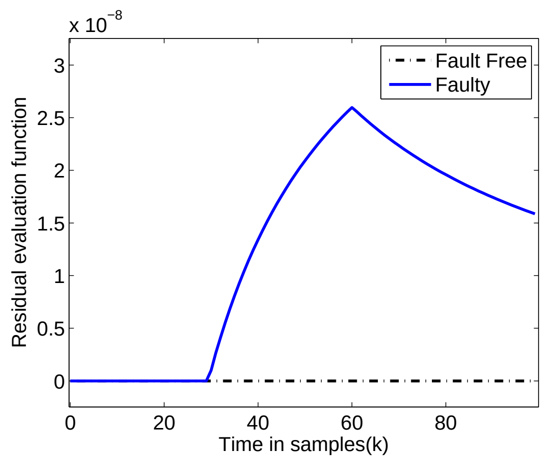

Fig. 5: Evaluation function of the nominal system with zero w_k.

For further comparison, we also simulate the system responses in the context of the event-triggered scheme parameter $\eta = 0.5$. In this case, Fig. 8 demonstrates the release instants of the event detector. In the simulation time, 45 times are triggered which is more than that in the context of $\eta = 0.2$. Fig. 9 shows the effectiveness of event-triggered scheme to the measurement output. In addition, Figs. 10 and 11 show the residual response and the residual evaluation function response with the disturbance in (32), respectively.

V. Conclusions

This paper has solved the fault detection problem for nonlinear networked systems under an event-triggered scheme. A polynomial event-triggered scheme has been first used to

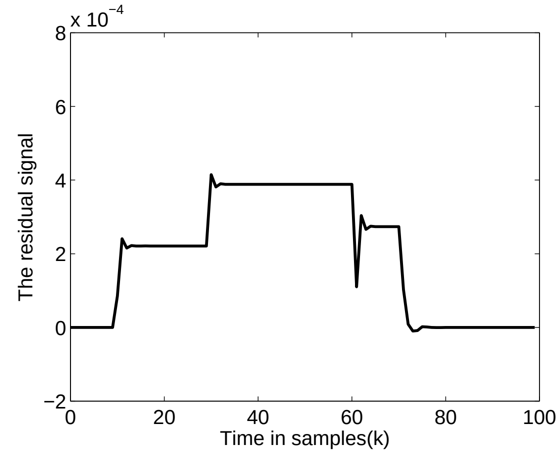

Fig. 6: Residual response of the nominal system with none-zero w_k.

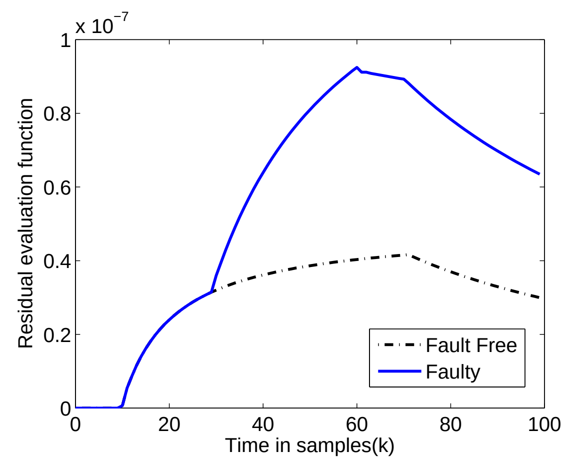

Fig. 7: Evaluation function of the nominal system with none-zero w_k.

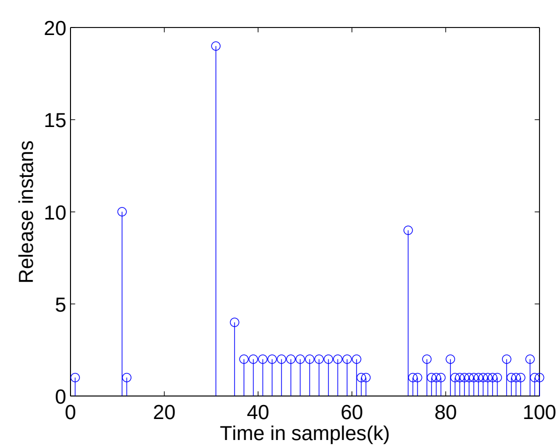

Fig. 8: The release instants with $\eta = 0.5$.

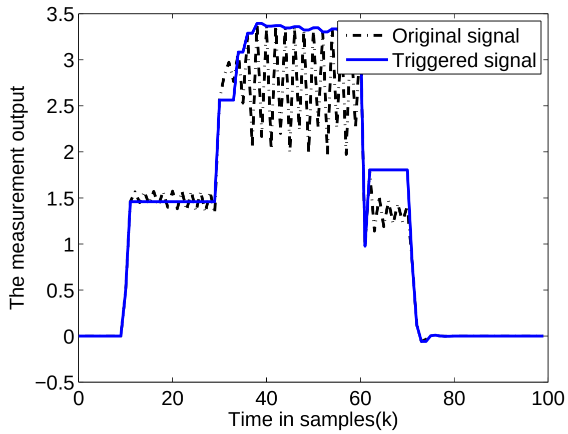

Fig. 9: The measured control output with $\eta = 0.5$.

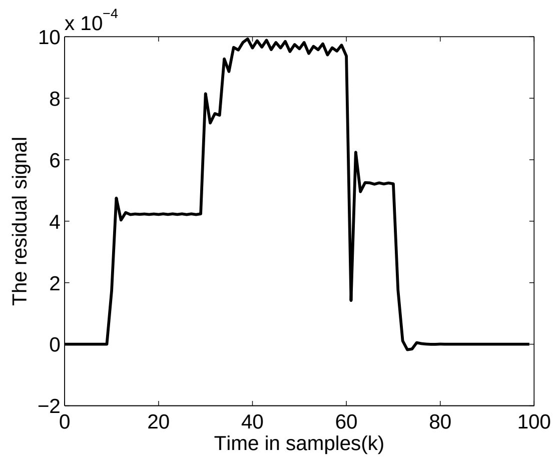

Fig. 10: Residual response of the nominal system with none-zero w_k and $\eta = 0.5$.

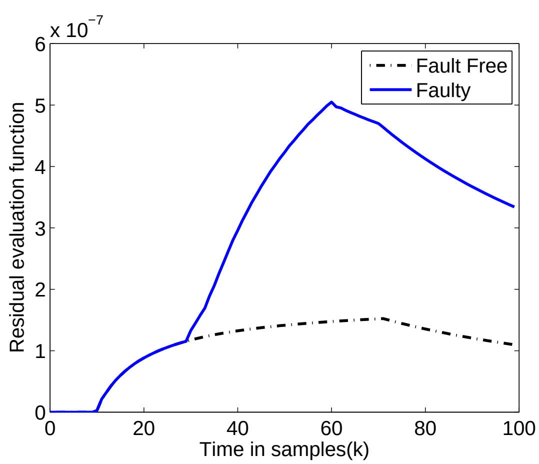

Fig. 11: Evaluation function of the nominal system with none-zero w_k and $\eta = 0.5$.

determine whether the signal should be transmitted or not. A novel polynomial fuzzy fault detection filter has been designed to guarantee the asymptotic stability of the residual system and satisfy the desired performance criteria. Sufficient conditions, which can be solved by the SOSTOOLS, have been represented as SOS. Simulation results have demonstrated the effectiveness of the proposed design scheme.

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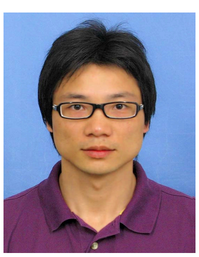

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